

Content Standards – Mathematics

What should students know and do? When should they learn it?

PRE-ALGEBRA

- Review place value, rounding, estimating, mentally calculating one-digit multiplication.
- Know and relate units of metric and customary measurement.
- Identify prime numbers, and calculate square numbers and square roots.
- Compare and order whole numbers, decimals, fractions, and integers.
- Calculate and solve problems by adding, subtracting, multiplying, and dividing whole numbers, integers, decimals, and fractions.
- Understand and use positive and negative exponents to solve problems.
- Simplify numerical expressions by applying the associative, commutative, and distributive properties, and order of operations.
- Solve one-step and two-step linear equations and inequalities.
- Use proportions to solve practical problems.
- Calculate percentages of numbers, and solve problems involving discounts, markups, unit price, simple interest, and tax.
- Graph linear functions; calculate and understand the slope of a line.
- Solve problems involving distance, rate, and time.
- Understand congruent and similar geometric figures.
- Use formulas to find perimeter and area of triangles, quadrilaterals, circles, and complex figures.
- Identify three-dimensional geometric figures and calculate surface area and volume.
- Use the Pythagorean Theorem to find lengths of missing sides of a right triangle.
- Display a set of data in an appropriate type of graph.
- Analyze data using mean, median, mode, and range.
- Represent possible outcomes for compound events in tables, grids, and tree diagrams to express the probability of each outcome.
- Express large and small numbers in scientific notation.
- Apply various problem solving strategies to solve word problems.
- Introduce and use algebraic terminology to solve problems.

ALGEBRA 1-2

- Identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers.
- Understand and use such operations as taking the opposite, finding reciprocal, and taking a root. Understand and use the rules of exponents.
- Write and solve equations and inequalities involving absolute values.
- Simplify expressions before solving linear equations and inequalities in one variable.
- Solve multi-step problems, including word problems, involving linear equations and linear inequalities in one variable.
- Graph linear equations and compute the x- and y-intercepts. Sketch the region defined by a linear inequality.
- Verify that a point lies on a line, given an equation of the line. Derive linear equations by using the point-slope formula.
- Understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Find the equation of a line perpendicular to a given line that passes through a given point.
- Solve a system of two linear equations in two variables algebraically and interpret the answer graphically. Solve a system of two linear inequalities in two variables and sketch the solution sets.

ALGEBRA 1-2, continued

- Add, subtract, multiply, and divide monomials and polynomials. Solve multi-step problems, including word problems, by using these techniques.
- Apply basic factoring techniques to polynomials.
- Understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions.
- Solve quadratic functions by factoring or completing the square, graph quadratic functions, and know that their roots are x-intercepts.
- Use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis.
- Use the quadratic formula to find the roots of a second-degree polynomial, to solve quadratic equations, and to solve physical problems, such as the motion of an object under the force of gravity.

GEOMETRY

- Demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning.
- Write geometric proofs, including proof by contradiction.
- Construct and judge the validity of a logical argument and give counterexamples to disprove a statement.
- Know the relationships of lines and segments, such as parallel, skew, intersecting, and perpendicular, and of angles, such as vertical, supplementary, complementary, and linear pairs.
- Know the basic theorems of congruence and similarity.
- Prove that triangles are congruent or similar, and use the concept of corresponding parts of congruent triangles.
- Know and use the triangle inequality theorem.
- Prove and use theorems involving the properties of parallel lines cut by a transversal and the properties of the various quadrilaterals.
- Find and use the measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems.
- Prove and solve problems regarding the properties of circles, their chords, secants, tangents, inscribed angles, and the inscribed and circumscribed polygons of circles.
- Know, derive and solve problems involving the perimeter, circumference, area, volume, and surface areas of common geometric figures.
- Compute the perimeters and areas of polygons.
- Compute the volumes and surface areas of solids and know how changes in dimensions affect these measures.
- Know how changes in dimensions affect the perimeter, area, and volume of geometric figures and solids.
- Use coordinate geometry, including the midpoint and distance formulas.
- Know and apply the Pythagorean Theorem to real world situations.
- Compute slopes of lines and know the relationship of the slopes of parallel and perpendicular lines.
- Graph and write the equations of lines, given a point and the slope or given two points.
- Know the basic definitions of right triangle trigonometry and solve for an unknown side or angle in a right triangle.
- Use the law of sines.
- Apply trigonometry to real world situations.
- Know the relationships of sides in the special right triangles (i.e., 30° - 60° - 90° and 45° - 45° - 90°).
- Draw and build various three-dimensional figures.
- Use lengths and areas to solve various geometric probabilities.
- Perform basic constructions.

ALGEBRA 3-4

- Solve equations and inequalities involving absolute value.
 - Solve systems of two and three or more linear equations and inequalities using various techniques.
 - Perform operations on polynomials.
 - Factor polynomials and recognize special forms.
 - Perform operations on complex numbers.
 - Perform operations on rational expressions including those with negative exponents in the denominator.
 - Solve quadratic equations by factoring, completing the square, or using the quadratic formula in both the real number system and the complex number system. Graph quadratic equations. Apply these techniques in solving word problems.
 - Demonstrate and explain the effect that changing a coefficient has on all families of graphs.
 - Graph quadratic functions and determine the intercepts and vertex.
 - Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
 - Know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay.
 - Understand and use the properties of logarithms including change of base to simplify logarithmic numeric expressions and identify their approximate values.
 - Demonstrate and explain how the geometry of the graph of a conic section depends on the coefficients of the quadratic equation representing it.
 - Use fundamental counting principles to compute combinations, permutations and probabilities.
 - Find the general term of arithmetic and geometric sequences.
 - Solve problems involving composition of functions and find the inverse of a function.
 - Solve optimization problems using linear programming techniques.
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TRIGONOMETRY

- Understand the relationship between radians and degrees, and convert between the two systems.
 - Know the definitions of the sine and cosine functions as the x and y coordinates of a point on the unit circle, and as the ratios of side lengths in a right triangle.
 - Know the definitions of the tangent, cotangent, secant, and cosecant functions.
 - Be able to state the domain and range of the six trigonometric functions, and be able to graph these six functions.
 - Understand and describe the amplitude, period, frequency, phase shift, and vertical shift of the six functions.
 - Be able to graph functions of the form $y = A \sin(Bx + C) + D$ and $y = A \cos(Bx + C) + D$.
 - Know the definitions and applications of the inverse trigonometric functions, and state their domain and range.
 - State the values of the six functions at various points on the unit circle (at certain angles) without a calculator.
 - Know and apply the angle addition, double-angle, and half-angle formulae for the sine, cosine and tangent functions.
 - Be able to verify or simplify trigonometric identities or expressions.
 - Use the properties of algebra and trigonometry to solve linear and quadratic trigonometric equations.
 - Know and use the law of sines and the law of cosines in a variety of situations.
 - Solve right and oblique triangles, and find the areas of triangles.
 - Graph complex numbers and polar coordinates, convert complex numbers from standard form to polar form (and back to standard form), and use DeMoivre's theorem to find the "n" roots of complex numbers.
 - Use a graphing calculator to explore, graph, and solve trigonometric functions and equations.
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PROBABILITY AND STATISTICS

- Know the definition of the notion of independent events and determine probabilities of particular events.
- Know the definition of conditional probability and determine probabilities of particular events.
- Use statistical terminology accurately.
- Collect, organize, and interpret data in standard ways, including pie charts, frequency tables, histograms, bar graphs, stem-and-leaf displays, scatter-plots, and box-and-whiskers plots.
- Determine and interpret information summarized by measures of central tendency.
- Determine and interpret information summarized by measures of position, like quartiles and percentiles.
- Compute and interpret the mean, variance, and the standard deviation of a distribution of data.
- Understand the concepts of a statistic of a distribution of values, the sampling distribution of a statistic, and the variability of a statistic.
- Find the line of best fit to a given distribution of data by using least squares regression.
- Compute and interpret the correlation coefficient of two variables and know the coefficient's properties.
- Use the line of best fit to determine predicted values of a continuous variable.
- Demonstrate an understanding of the notion of discrete and continuous random variables.
- Calculate probabilities of events explained by the normal and the standard normal distribution.
- Know the central limit theorem and use it to obtain approximations for probabilities that are distributed binomially.
- Estimate single population means and single proportions (P-value) with confidence intervals.
- Estimate the difference of two means and two proportions with confidence intervals.
- Determine the sample size necessary to obtain a predetermined margin of error when estimating with confidence intervals.
- Test hypotheses about a single population mean (Z-test and T-test) and a single population proportion.
- Test hypotheses about the difference of two population means and two population proportions.
- Analyze common misuses of linear regression and correlation.
- Distinguish between correlation and causation.
- Analyze the assumed distribution of one categorical variable using the chi square distribution.
- Analyze the association of two categorical variables using the chi square distribution.

ADVANCED TOPICS

- Understand the relationship between Cartesian, polar, and parametric equations, and to translate between these systems.
- Work with complex numbers symbolically, numerically, and graphically, and express solutions to equations in any of these ways.
- Identify and describe a quadratic equation in two variables by putting the equation into standard conic form.
- Identify and graph a rational function by finding the roots and asymptotes.
- Find and describe the limit of a sequence or a function as the independent variable approaches a number or infinity.
- Solve a system of linear or quadratic equations algebraically, and interpret the solution(s) graphically.
- Perform matrix operations and use matrices to solve systems of linear equations.
- Work with trigonometric and inverse trigonometric functions in a variety of situations, both circular and triangular.
- Analyze and write equations to model data, including first-, second-, and third-degree functions, exponential functions, and trigonometric functions.
- Understand and apply parametric equations and vectors to a variety of appropriate situations, and be able to find and explain the dot- and cross-products of vectors.
- Use exponential and logarithmic equations to model growth, decay, and compound interest.

ADVANCED TOPICS, *continued*

- Define and describe geometric and arithmetic sequences and series, and apply sigma notation and the concept of convergence and divergence.
- Explore and describe higher-order polynomial equations, and describe the number and nature of the roots.
- Use a graphing calculator effectively in both numerical and graph analysis of functions.
- Understand and apply the basic theories of combinatorics and probability.
- Apply the basic concepts of statistics to appropriate problems, including correlation, standard deviation, and different types of regression.
- Give proofs of various formulae using mathematical induction, and know and apply the fundamental theorem of algebra.

CALCULUS

- Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Know the definitions of convergence and divergence of a function as the domain variable approaches either a number or infinity.
- Use theorems evaluating the limits of sums, products, quotients, and composition of functions.
- Use graphing calculators to verify and estimate limits.
- Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.
- Demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability.
- Demonstrate an understanding of the interpretation of the derivative of a function as the slope of the tangent line to the graph of the function.
- Demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change; use derivatives to solve a variety of problems from physics, chemistry, economics, and other disciplines that involve the rate of change of a function.
- Derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.
- Know the product, quotient, and chain rules and apply them to a variety of composite functions.
- Find and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and other disciplines.
- Compare derivatives of higher orders.
- Know and apply L'Hôpital's rule.
- Use differentiation to sketch, by hand, graphs of functions and identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.
- Know Newton's method for approximating the zeros of a function.
- Use differentiation to solve related rate problems in a variety of pure and applied contexts.
- Use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.
- Know the definition of the definite integral by using Riemann sums and use this definition to approximate integrals using right hand sums and left hand sums both by hand and calculator.
- Apply the definition of the integral to model problems in physics, economics, and other disciplines, obtaining results in terms of integrals.
- Demonstrate knowledge of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.
- Use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, density, supply and demand, and work.
- Compute, by hand, the integrals of a wide variety of functions using techniques of integration, such as substitution, integration by parts, partial fractions, completing the square, and by table.
- Use the algorithms involved in the midpoint rule, the trapezoidal rule, Simpson's rule, and Newton's method to approximate integrals numerically both by hand and calculator.

CALCULUS, continued

- Understand improper integrals as limits of definite integrals.
- Demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. Use such tests as the comparison test and ratio test to determine whether a series converges.
- Understand and compute the radius (interval) of convergence of power series.
- Differentiate and integrate the terms of a power series in order to form new series from known ones.
- Calculate Taylor polynomials and Taylor series of basic functions.
- Know the techniques of solution of selected elementary differential equations and their applications to a wide variety of situations, including growth- and decay-problems (Newton's Law of Cooling).

These state standards have been established by the California Board of Education and adopted by the Santa Barbara School Districts Board of Education. A complete set of standards is available at the District Office.

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